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# Design, Modelling and Validation of a Linear Joule Engine Generator designed for Renewable Energy Sources

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## Abstract

The Linear Joule Engine Generator (LJEG) incorporates the Joule Engine technology and the permanent magnet linear alternator design, which is a promising power generation device for the applications of range extenders for electric vehicles, Combined Heat and Power (CHP) systems, or as a stand-alone power unit. It combines the advantages from both a Joule Engine and a linear alternator, *i.e.* high efficiency, compact in size, and flexible to renewable energy integration, etc. In this paper, the background and recent developments of the LJEGs are summarised. A detailed 0-dimensional numerical model is described for the evaluation of the system dynamics and thermodynamic characteristics. Model validation is conducted using the test data obtained from both a reciprocating Joule Engine and a LJEG prototype, which proved to be in good agreement with the simulation results. The fundamental operational characteristics of the system were then explained using the validated numerical model. It was found that the piston displacement profile has certain similarity with a sinusoidal wave function with an amplitude of 51.0 mm and a frequency of 13 Hz. The electric power output from the linear alternator can reach 4.4kW<sub>e</sub>. The engine thermal efficiency can reach above 34%, with an electric generating efficiency of 30%.

**Keywords:** Linear Joule-cycle Engine; linear expander; linear alternator; numerical model; model validation.

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## Nomenclature

$A_{com}$ (m <sup>3</sup> )	compressor piston area	$p_{com\_in}$ (Pa)	intake gas pressure of compressor
$A_{exp}$ (m <sup>3</sup> )	expander piston area	$P_e$ (W)	electric power output of alternator
$A_{exp\_surf}$ (m <sup>2</sup> )	surface are in contact with gas	$P_{ex}$ (W)	indicated power of the linear expander
$A_d$ (m <sup>2</sup> )	reference area of the flow	$\dot{Q}_{ht}$ (J/s)	heat flow rate between cylinder wall and gas
$C_d$ (-)	discharge coefficient	$p_d$ (Pa)	downstream air pressure
$C_e$ (N/(m·s <sup>-1</sup> ))	load constant of alternator	$p_{exp}$ (pa)	pressure in linear expander
$C_k$ (-)	kinetic friction coefficient	$p_{exp\_l}$ (Pa)	pressure from left chamber of expander
$C_s$ (-)	static friction coefficient	$p_{exp\_r}$ (Pa)	pressure from right expander
$\dot{m}_{flow}$ (kg/s)	mass flow rate through valve	$p_{exp\_in}$ (Pa)	intake gas pressure of linear expander
$\dot{m}_{expi}$ (m/s)	mass flow rate in/out of the valve	$R$ ( $\Omega$ )	resistance of the circuit
$\overrightarrow{F_{exp}}$ (N)	pressure force from linear expander	$R_s$ ( $\Omega$ )	internal resistance
$\overrightarrow{F_{exp\_l}}$ (N)	pressure force from left expander	$R_L$ ( $\Omega$ )	resistance of the external load
$\overrightarrow{F_{exp\_r}}$ (N)	pressure force from right expander	$T_u$ (K)	temperature of upstream
$\overrightarrow{F_{com}}$ (N)	pressure force from linear compressor	$T_w$ (K)	average surface temperature of cylinder wall
$\overrightarrow{F_{com\_l}}$ (N)	pressure force from left compressor	$v$ (m/s)	piston velocity
$\overrightarrow{F_{com\_r}}$ (N)	pressure force from right compressor	$v_p$ (m/s)	average piston speed
$\overrightarrow{F_e}$ (N)	resistance force from alternator	$V$ (m <sup>3</sup> )	instantaneous cylinder volume
$\overrightarrow{F_f}$ (N)	frictional force	$V_{com}$ (m <sup>3</sup> )	working volume of linear compressor
$i$ (A)	current in the circuit	$V_{exp}$ (m <sup>3</sup> )	working volume of linear expander
$p_{com}$ (Pa)	pressure in the compressor	$x$ (m)	piston displacement
$p_{com\_l}$ (Pa)	pressure from left of compressor	$\gamma$ (-)	heat capacity ratio
$p_{com\_r}$ (Pa)	pressure from right compressor	$\varepsilon$ (V)	electromotive voltage

25

## 26 1. Introduction

27 The Linear Joule Engine Generator (LJEG) is derived from the Joule Engine technology and  
28 incorporates a permanent magnet in a linear alternator design. The Joule Engine technology uses a free  
29 piston configuration with a potential high efficiency due to its mechanical simplicity and minimal

frictional loss, in addition it employs an external (out-of-cylinder) heat addition method to adapt to various renewable energy sources [1-3]. The permanent magnet linear alternator is reported to be compact in size, and efficient in electricity generation [4-7]. The LJEG takes advantages of both a Joule Engine and the Linear Engine Generator, and it provides an alternative high-efficiency, renewable energy adaptive, prime mover for transportation and power generation applications. At the same time, it offers flexibility at a time when it is expected to see a major increase in the low-carbon/carbon-free fuel variety, e.g. biogas, biofuels, hydrogen and ammonia, in these sectors towards 2050.

### 1.1 Joule Engine technology

The Joule cycle (or Brayton Cycle) is widely employed in gas turbines, where air intake is compressed, before fuel is burnt under constant pressure, and then, the exhaust gas expands out to ambient pressure. Typically the compression and expansion processes are performed by turbomachinery [8]. In theory it has isobaric heat addition and heat rejection processes, and isentropic compression and expansion. The reciprocating Joule Engine technology applies split a reciprocating compressor and expander to improve its efficiency, which was proposed as an engine for application in the micro CHP systems [1, 3, 9].

Moss *et al.* estimated the performance of a Joule Engine in small size (1-10 kW) with a simple simulation model in Matlab [1]. M. Alaphilippe, *et al.* provided a theoretical investigation on the coupling of a two-stage parabolic trough solar concentrator with a hot air Joule Engine [10]. The preliminary results were reported to be promising of coupling a simple parabolic trough and a Joule Engine. Wojewoda and Kazimierski provided investigation on operation of an externally heated valve Joule Engine [11]. A numerical model was presented, and the heat exchanger operation was further investigated. M. Creytx, *et al.* developed a numerical model of an open cycle Joule Engine, which was focused on the thermodynamic aspects [12]. The reported system thermodynamic efficiency was 37%

54 after some optimisation work. Bell and Partridge presented a first-order model of a Joule Engine, and  
55 the model included combustion, clearance volume, gas leakage, pressure drop, and friction [2].  
56 Another system was reported by the researchers at Plymouth University, the system power output and  
57 efficiency were simulated, indicating an engine thermal efficiency of up to 33% [2]. The model  
58 validation was performed using the testing results of both a demonstration engine and a prototype  
59 engine [13].

## 60 **1.2 Linear Engine Generator technology**

61 The Linear Engine Generator is linear ‘crank-less’ power device that couples a linear internal  
62 combustion engine with a linear electric generator, it uses conventional diesel or Otto cycles [4, 14,  
63 15]. The piston of the engine is connected with the translator of the generator. Combustion takes place  
64 in the engine cylinder, and the high pressure gas during the expansion process is used to drive the  
65 piston and the translator, and the linear generator produces electricity [16]. There have been different  
66 prototypes reported by different research groups [17-23]. Successful implementations of single cylinder  
67 Linear Engine Generators have been reported by Toyota Central R&D Labs Inc. and the German  
68 Aerospace Centre (DLR), which were both composed of a single cylinder engine, a linear electric  
69 generator, and a gas spring rebound chamber [23-26]. For the prototype developed at DLR, it was  
70 operated at 21 Hz, with an electric power output of approximately 10 kW [27]. The TDC achieve was  
71 found to be at 57.5% of the periodic time [28]. For the dual-piston dual-cylinder Linear Engine  
72 Generator, several prototypes have been designed in Beijing Institute of Technology [6, 7]. Both 0/1  
73 dimensional and multi-dimensional simulation were undertaken to predict the dynamic and  
74 thermodynamic performance of the system [29-31]. Successful engine cold start-up has been reported,  
75 and the combustion took place when the cylinder pressure reached the required level for ignition [7,  
76 32, 33]. The piston was controlled to oscillate between two set positions with constant speed [34, 35].  
77 The predicted system efficiency was around 35%. The potential disturbances to the system were  
78 analysed, and a cascade control strategy was proposed for the piston stable control [36, 37].

### 79    **1.3 Linear Joule Engine Generator development**

80    The Linear Joule Engine Generator concept was first proposed by the authors' group, initially aiming  
81    for application for micro-scale CHP generation [3]. Simple calculations were undertaken, and the  
82    simulation results suggested that a domestic CHP plant based on the proposed technology could reach  
83    an electric generating efficiency of above 30%. With a heating temperature of around 1100 K and a  
84    compressor outlet pressure of 6 bar, the engine could produce 4.5 kW of mechanical power. Whilst,  
85    through waste heat recovery technology, the total system could reach a promising efficiency of over  
86    90%. Later on, a 3-dimensional diagram of the proposed LJEG system was presented by the authors  
87    [9]. The geometry parameters of the system were optimised in LMS AMESim software, which  
88    provided a solid basis for the manufacturing of the prototype. Meanwhile, Wu et al. presented a  
89    coupled dynamic model of the Linear Joule Engine and the connected permanent magnet linear electric  
90    generator, aiming to provide a better prediction of the system performance. It was estimated that the  
91    LJEG system could generate 1.8 kW electricity, with an engine thermal efficiency of 34% and electric  
92    generating efficiency of 30% [38].

### 93    **1.4 Aims and methodology**

94    In this research, the background and recent developments of the LJEG are summarised. A more  
95    detailed numerical model of the system will be described, which includes the sub-models for the piston  
96    dynamics, the reactor, the linear expander, the linear compressor, and the linear generator, etc. The  
97    model validation will be performed with the testing data from both a reciprocating Joule Engine, and  
98    a LJEG prototype developed by the authors' group. The system dynamics and thermodynamics  
99    characters will be identified with the validated model.

## 2. System configuration

For an ideal Joule Engine Cycle (as illustrated in Figure 1), it usually consists of four processes, *i.e.* adiabatic compression process in the compressor, constant pressure fuel combustion process, adiabatic expansion process in the expander [39]. It should be noted that the “Combustor” shown in Figure 1 can be replaced with any fuel combustion, waste heat, or renewable energy reaching certain temperature, and the gas will drive the expander.

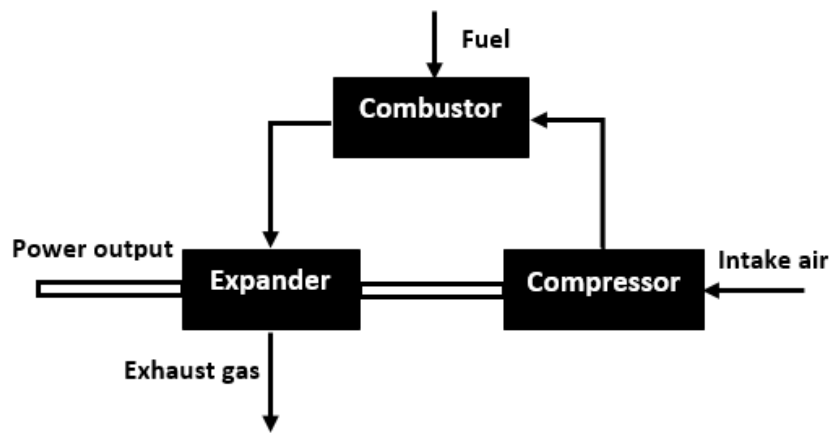


Figure 1. System schematic figure of a Joule Engine Cycle

The configuration of the LJEG prototype developed by the authors is illustrated in Figure 2, using an external reactor to burn fuel as heat input. It is an open system, and the exhaust gas after the expander would be high-pressure, high temperature gas. The air is compressed in a positive displacement compressor featured with a double-acting free piston and several poppet valves for intake and discharge; the compression of the air results in a high pressure, high temperature air, which is fed into an external reactor. The fuel is fed into the reactor and reacts with the air to produce heat and high pressure gas. The expander reduces the pressure and temperature by expanding the working fluid and this expansion is used to drive the linear generator and the compressor.



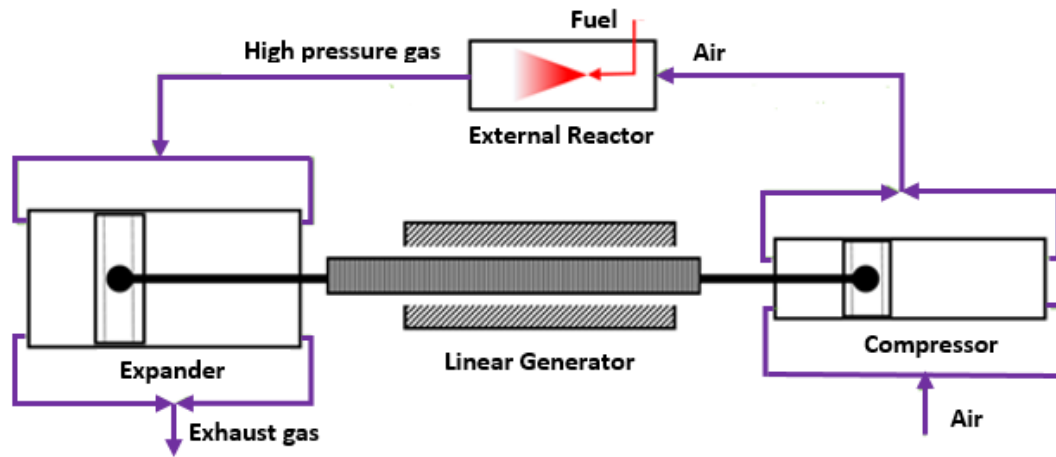


Figure 2. The LJEG prototype configuration with a reactor as an example heat input

### 3. Numerical modelling

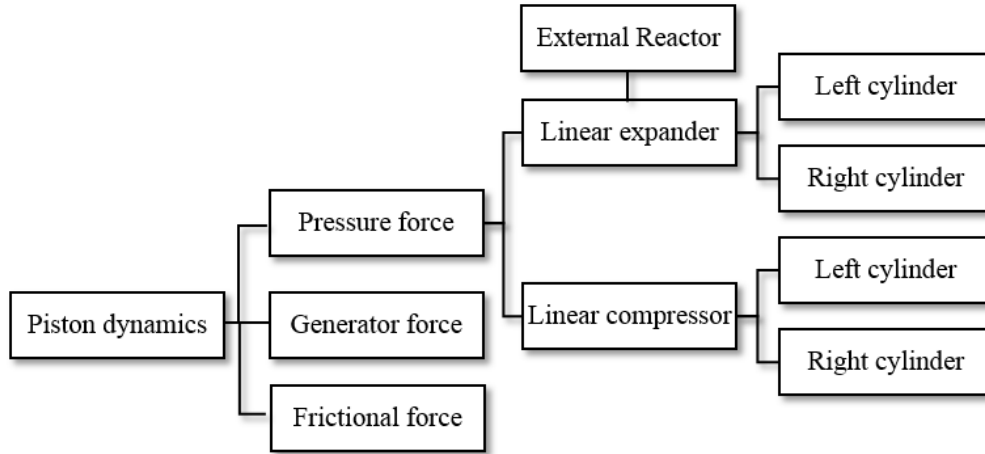
#### 3.1 Model structure

The numerical model aims to describe the dynamic and thermodynamic characteristics of the LJEG system, e.g. the piston motion, the pressure variation in the expander and the compressor, the system power output, the system efficiency, etc. As the piston in the proposed system is not restricted by a mechanical linkage, the piston motion is determined by the forces acting on it, which are the gas pressure forces from the linear expander and the compressor, the resistance force from the linear generator, the frictional force, and the inertia of the moving mass. Therefore a piston dynamic model is developed on the top level. The structure of the numerical model is demonstrated in Figure 3.

Three sub-model that describe the specific forces that acting on the pistons are developed on a lower level, and the calculated forces are used as feedback signals to the top-level piston dynamic model to determine the piston acceleration. The pressure forces are determined by the gas thermodynamic processes from both chambers of the linear expander and the linear compressor, which consider the compression/expansion of the piston, gas intake/exhaust through the valves, the heat transfer from the

132 gas of the chamber to the wall, etc. During the operation of the system, the linear generator will  
 133 generate electricity, and outputs an electric resistance force acting on the piston.

134 The performance of the linear expander is affected by the high-pressure, high-temperature gas from  
 135 the external reactor, which is the intake gas to the chambers of the expander through the intake valves.  
 136 The outputs of the reactor, *i.e.* the gas pressure, temperature, and mass flow rate, will be used as input  
 137 parameters to the linear expander during the gas intake process. The external reactor can be replaced  
 138 with any fuel combustor, solar energy, waste heat, or renewable energy that can drive the expander.  
 139 The external reactor would largely follow the isobaric heat addition process with a confined pressure  
 140 fluctuation regardless fuel species, thus the inlet pressure and temperature of the linear expander are  
 141 assumed to be constant.



142

143 Figure 3. The structure of the numerical model

### 144 3.2 Piston dynamics model

145 The forces acting on the pistons are the gas pressure forces from the linear expander and the  
 146 compressor, the resistance force from the linear generator, the frictional force, and the inertia of the  
 147 moving mass, which can be expressed as blow according to the Newton's Second Law:

$$148 \quad \vec{F}_{exp} + \vec{F}_{com} + \vec{F}_e + \vec{F}_f = m\ddot{x} \quad (1)$$

$$\overrightarrow{F_{exp}} = \overrightarrow{F_{exp\_l}} + \overrightarrow{F_{exp\_r}} \quad (2)$$

$$\overrightarrow{F_{com}} = \overrightarrow{F_{com\_l}} + \overrightarrow{F_{com\_r}} \quad (3)$$

Where  $\overrightarrow{F_{exp}}$  (N) is the pressure force from the linear expander;  $\overrightarrow{F_{exp\_l}}$  (N) is the pressure force from the left chamber of the linear expander;  $\overrightarrow{F_{exp\_r}}$  (N) is the pressure force from the right chamber of the linear expander;  $\overrightarrow{F_{com}}$  (N) is the pressure force from the linear compressor;  $\overrightarrow{F_{com\_l}}$  (N) is the pressure force from the left chamber of the linear compressor;  $\overrightarrow{F_{com\_r}}$  (N) is the pressure force from the right chamber of the compressor;  $\overrightarrow{F_e}$  (N) is the resistance force from the linear electric alternator;  $\overrightarrow{F_f}$  (N) is the frictional force.

The gas forces from both chambers of the linear expander and compressor can be calculated by the gas pressure and piston effective area, where can be represented as following:

$$\overrightarrow{F_{exp\_l}} = p_{exp\_l} \cdot A_{exp} \quad (4)$$

$$\overrightarrow{F_{exp\_r}} = p_{exp\_r} \cdot A_{exp} \quad (5)$$

$$\overrightarrow{F_{com\_l}} = p_{com\_l} \cdot A_{com} \quad (6)$$

$$\overrightarrow{F_{com\_r}} = p_{com\_r} \cdot A_{com} \quad (7)$$

Where  $p_{exp\_l}$  (Pa) is the cylinder pressure from the left chamber of the linear expander;  $p_{exp\_r}$  (Pa) is the cylinder pressure from the right chamber of the linear expander;  $p_{com\_l}$  (Pa) is the cylinder pressure from the left chamber of the linear compressor;  $p_{com\_r}$  (Pa) is the cylinder pressure from the right chamber of the linear compressor;  $A_{exp}$  (m<sup>2</sup>) is the piston area of the expander;  $A_{com}$  (m<sup>2</sup>) is the piston area of the compressor.

### 3.3 Linear expander

The thermodynamic processes in a chamber of the linear expander mainly include the compression/expansion process due to the piston movement, heat transfer from gas in the chamber to the wall, as well as the inlet and exhaust gas exchange processes. By applying the first law of thermodynamics on the charge in the chamber and ideal gas equation, yields the pressure calculation equation for one of the two chambers (detailed derivation process can be found in the previous publications of the authors [25]):

$$\frac{dp_{exp}}{dt} = \frac{\gamma-1}{V_{exp}} \left( -\frac{dQ_{ht}}{dt} \right) - \frac{p_{exp}\gamma}{V_{exp}} \frac{dV_{exp}}{dt} + \frac{\gamma-1}{V_{exp}} \sum_i \dot{m}_{expi} h_{expi} \quad (8)$$

Where  $p_{exp}$  is the pressure in the chamber of the linear expander (pa);  $\gamma$  is the heat capacity ratio;  $V_{exp}$  is the working volume of the linear expander for one cylinder ( $m^3$ );  $\dot{m}_{expi}$  is the mass flow rate in or out of the valve (m/s);  $h_{expi}$  is the specific enthalpy of the mass flow ( $J \cdot kg^{-1}$ ).

The heat transfer between the walls and the gas of one chamber of the expander is modelled according to Hohenber [40]:

$$\dot{Q}_{ht} = 130V^{-0.06} \left( \frac{p(t)}{10^5} \right)^{0.8} T^{-0.4} (v_p + 1.4)^{0.8} \cdot A_{exp\_surf} (T - T_w) \quad (9)$$

Where  $\dot{Q}_{ht}$  is heat flow rate (J/s);  $V$  is the instantaneous cylinder volume ( $m^3$ );  $v_p$  is the average piston speed (m/s),  $A_{exp\_surf}$  ( $m^2$ ) is area of the surface in contact with the gas in the chamber of the expander;  $T_w$  (K) is the average surface temperature of the cylinder wall.

The mass flow rate through the valves,  $\dot{m}_{flow}$  is assumed to be represented by a compressible flow through a flow restriction. It is determined by temperature, composition, the gas pressure, and a reference area of the valve [24], which is given by:

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$$\dot{m}_{flow} = \begin{cases} \frac{C_d A_d p_u}{(RT_u)^{1/2}} \left(\frac{p_d}{p_u}\right)^{\frac{1}{\gamma}} \sqrt{\frac{2\gamma}{\gamma-1} \left[1 - \left(\frac{p_d}{p_u}\right)^{\frac{(\gamma-1)}{\gamma}}\right]}, & p_d/p_u > [2/(\gamma+1)]^{\gamma/(\gamma-1)} \\ \frac{C_d A_d p_u}{(RT_u)^{1/2}} \gamma^{1/2} \left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/2(\gamma-1)}, & p_d/p_u \leq [2/(\gamma+1)]^{\gamma/(\gamma-1)} \end{cases} \quad (10)$$

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Where  $\dot{m}_{flow}$  is the mass flow rate through a poppet valve (kg/s);  $C_d$  is the discharge coefficient;  $A_d$  is the reference area of the flow ( $m^2$ );  $T_u$  is the temperature of the upstream of the flow restriction (K);  $p_u$  is the pressure of the upstream of the flow restriction (Pa);  $p_d$  represents the downstream air pressure of the flow restriction (Pa).

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### 3.4 Linear compressor

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For ideal gas, both compression and expansion process are governed by the gas pressure and its volume after the intake valve and exhaust valve closed. The air leakage across the piston rings was considered negligible, hence it is assumed that the gas is completely isolated by the piston rings and there is no air mass transfer. The relationship between gas pressure  $p_{com}$  and volume of the chamber  $V_{com}$  during the compression/expansion process is listed below:

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$$\frac{dp_{com}}{dt} = \frac{\gamma-1}{V_{com}} \left( -\frac{dQ_{ht}}{dt} \right) - \frac{p_{com}\gamma}{V_{com}} \frac{dV_{com}}{dt} \quad (11)$$

200

201

Where  $p_{com}$  is the pressure in the chamber of the linear compressor (pa);  $\gamma$  is the heat capacity ratio;  $V_{com}$  is the working volume of the linear compressor for one cylinder ( $m^3$ ).

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The intake and exhaust valves here adopted are reed valves, which open when the pressure of the upstream is higher than that of the downstream. When the intake valve is opened, the gas pressure in the chamber of the linear compressor is assumed to be equal with the intake pressure immediately; and the gas pressure is assumed to be same with the exhaust pressure (or the intake pressure to the linear expander) once after the exhaust valve is open. In summary, the gas pressure in one chamber of the linear compressor is described by:

$$p_{com\_2} = \begin{cases} p_{exp\_in}; & p_{com\_2} > p_{exp\_in} \\ p_{com\_1}(V_{com\_1}^\gamma/V_{com\_2}^\gamma); & p_{com\_in} < p_{com\_2} < p_{exp\_in} \\ p_{com\_in}; & p_{com\_2} < p_{com\_in} \end{cases} \quad (12)$$

Where  $p_{com\_in}$  (Pa) is the intake gas pressure of the linear compressor;  $p_{exp\_in}$  (Pa) is the intake gas pressure of the linear expander, which is the same with the exhaust gas pressure of the linear compressor.

### 3.5 Linear electric generator

The linear electric machine is operated as a generator, electrical current is drawn from the alternator coils through the continuous back and forth movement of the mover. The linear generator is modelled using a simplified numerical model to make it feasible with limited amount of design parameters known to the users. Figure 4 illustrates an equivalent circuit of the linear electric machine.

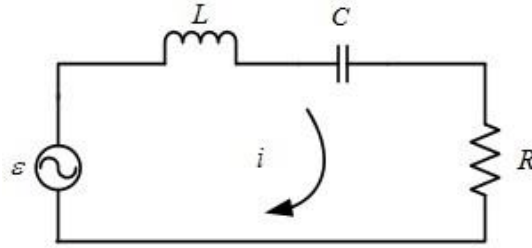


Figure 4 Equivalent circuit of the linear electric machine [31]

Then the Faraday's electromagnetic induction laws give the electromotive voltage  $\varepsilon$  (V) as

$$\varepsilon(t) = -N \frac{d\phi}{dt} = -K_v \frac{dx}{dt} = -K_v \cdot v \quad (13)$$

Where  $\phi$  is the magnetic flux;  $K_v$  is a motor property and determined by the design parameters of the motor and can be found in the manual;  $x$  is the piston displacement (m);  $v$  is the piston velocity (m/s).

The induced current is determined by the voltage and the load circuit, assuming the load circuit is purely resistive ( $C = 0, L = 0$ ), it can be derived by:

$$\varepsilon(t) = (R_s + R_L)i(t) \quad (14)$$

226 Where  $R$  is the resistance of the circuit ( $\Omega$ ),  $R_S$  is the internal resistance ( $\Omega$ ), and  $R_L$  is the resistance of  
227 the external load ( $\Omega$ );  $i$  is the current (A).

228 Then the current in the coil is then expressed by:

$$229 \quad i(t) = -\frac{K_v}{R_S + R_L} \cdot v \quad (15)$$

230 As the load force of the electric machine is assumed to be proportional to the current of the circuit  
231 according to electromagnetic theory, the resistance force from the alternator is then written as:

$$232 \quad F_e = -C_e v \quad (16)$$

233 Where  $C_e$  is the load constant of the alternator ( $\text{N}/(\text{m} \cdot \text{s}^{-1})$ ), which can be calculated from the physical  
234 parameters of the alternator design specifications.

### 235 **3.6 Frictional force**

236 An analysis of engine friction mechanisms in four stroke spark ignition and diesel engines is presented  
237 by Heywood [41]. Friction work is expected to be lower than conventional internal combustion engines  
238 due to the elimination of the crank mechanism. Thus the friction in the wrist pin, big end, crankshaft,  
239 camshaft bearings, the valve mechanism, gears, or pulleys and belts which drive the camshaft and  
240 engine accessories have been removed. The total friction force  $F_f$  of each piston is estimated as a linear  
241 combination of piston velocity plus a constant  $C_s$ , as shown in the equation below [42]:

$$242 \quad F_f = -(C_k \cdot |v| + C_s) \cdot \text{sign}(v) \quad (17)$$

243  $C_k$  is the kinetic friction coefficient related to the instantaneous velocity, and the  $C_s$  is the static  
244 friction coefficient as a constant part of the frictional force.

## 245 4. Model implementation and validation

### 246 4.1 Simulation model implementation

247 The simulation model is developed in Matlab/Simulink. The design parameters of the model are  
248 derived from the preliminary design of the prototype in built/testing and the initial boundary conditions  
249 are defined based on the practical starting conditions and the assumptions made in the model  
250 mentioned above. Both the piston displacement and velocity generated in the simulation are monitored  
251 and fed back to a controller which imposes the valve timings. The initial piston position is assumed to  
252 be at its TDC (approximately 8 mm from the cylinder head) in the left chamber of the linear expander.  
253 The prototype specifications and the values of the input parameters for the system operation cycles are  
254 listed in Table 1. The system design parameters and the input boundary parameters will be further  
255 optimised at the next stage. The inlet pressure of the reactor is set to be the same with the outlet pressure  
256 of the linear compressor, and the inlet pressure of the linear expander, which can be adjusted during  
257 the simulation. The outlet pressure of the linear compressor, and the mass flow rate to the reactor are  
258 all variables, which will affect the inlet pressure to the reactor and the linear expander, and the intake  
259 temperature of the linear expander correspondingly.

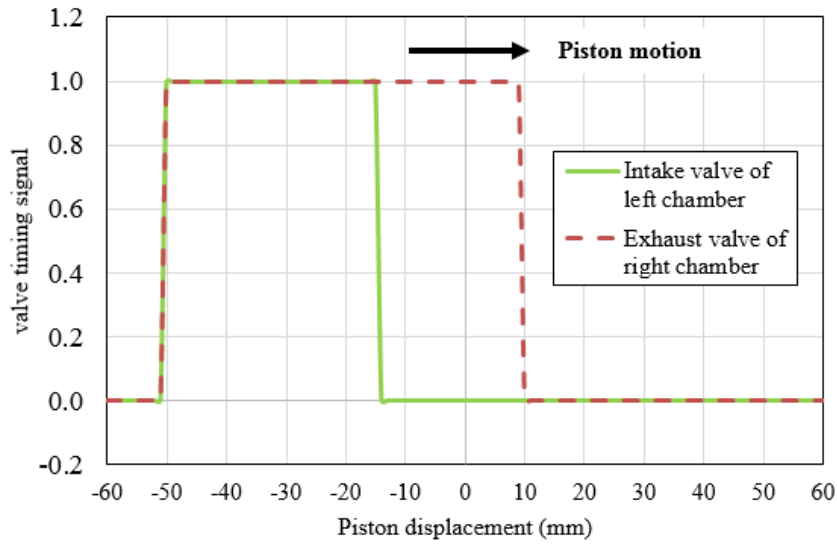
Components	Parameters [Unit]	Value
Expander	Moving mass [kg]	8.5
	Maximum stroke [mm]	120.0
	Effective bore [mm]	80.0
	Inlet pressure [bar]	7.0
	Inlet temperature [K]	1100.0
	Valve number	4
	Valve diameter [mm]	32.5
	Valve lift [mm]	8.13
Linear compressor	Maximum stroke [mm]	120.0



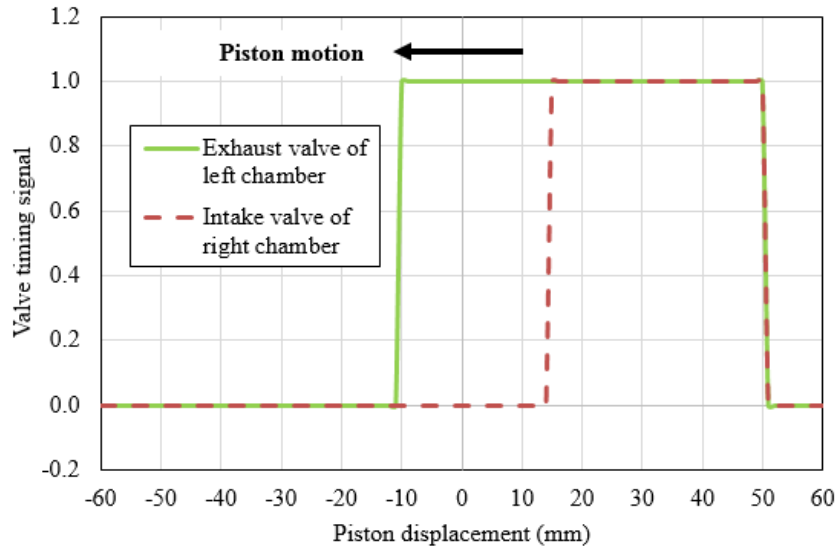
	Effective bore [mm]	66.0
	Inlet pressure [bar]	1.0
	Outlet pressure [bar]	7.0
Linear generator	load constant [N/m·s <sup>-1</sup> ]	367.6

Table 1. Prototype specifications and input parameters

As the valves are actuated based on the piston position, the scavenging durations will be significantly affected by the piston speed and profile. The step functions are used to impose the valve-lift profiles, as which proved to be aligned with the response of the installed valve system. The opening and closing valve timings can be adjusted via the controller to optimise the scavenging process. The expansion process of the expander is initialised after the intake valve open (IVO), which is actuated when the piston reaches its TDC. The exhaust valve open (EVO) is triggered when the piston reaches its BDC. The valve timings versus the piston displacement for both chambers of the expander are illustrated in Figure 5, and example piston dead centres are set to -50 mm and 50 mm.



(a) Rightward stroke



(b) Leftward stroke

Figure 5. The example valve timings for both chambers of the linear expander

#### 4.2 Validation with a Reciprocating Joule Engine

The simulation results from the model were first compared to data from a Reciprocating Joule Engine developed at University of Plymouth [8]. The configuration of a Reciprocating Joule Engine is different from the LJEG system, which is illustrated in Figure 6. The comparison was undertaken to verify that the simulation developed in this research produces the realistic results and is valid for predicting the prototype performance in different system operation conditions. The system specifications were set to be identical with the Reciprocating Joule Engine introduced in [8]. The Reciprocating Joule Engine input parameters are listed in Table 2, the bores of the expander and compressor are the same.

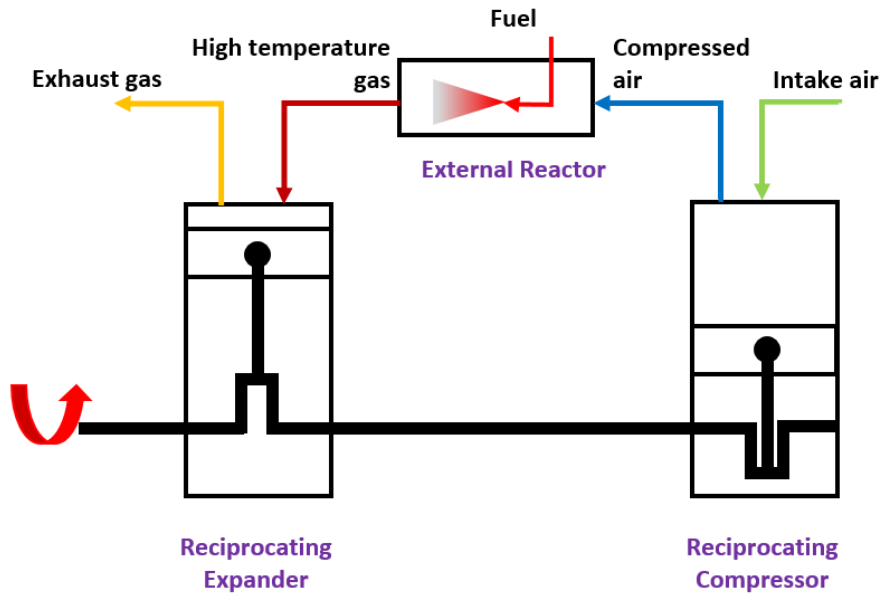


Figure 6. Schematic configuration of a Reciprocating Joule Engine

Parameter [Unit]	Value
Stroke [mm]	61.5
Bore [mm]	82.0
Clearance volume [cc]	30
Supply pressure [bar]	7.5
Supply temperature [K]	850

Table 2. The Reciprocating Joule Engine specification for model validation [8]

During the testing, the engine was operated on external compressed air (with no compressor connected). The test data and simulation results of the expander pressure are compared in Figure 7. For the Reciprocating Joule Engine, the expander was operated on external compressed air, and the compressor was not connected, which would contribute to the difference with the simulation results. The valve timing was set based on the crank angle, and the inlet valve was set to open at  $10^\circ$  before TDC, and close at  $80^\circ$  after TDC. The exhaust valve was set to open at  $10^\circ$  before BDC, and close at  $70^\circ$  before TDC [8]. As for the LJEG concept used in this research, the piston is not restricted with mechanical crankshaft linkage, and the piston movement cannot be represented with crank angle as

the Reciprocating Joule Engine does. As a result, the setting of the valve timing in the simulation would not be exactly the same with the test engine, which introduces the error to a great extent. Despite of the errors, the numerical model can simulate the performance of the expander, and predict the variation of the cylinder pressure.

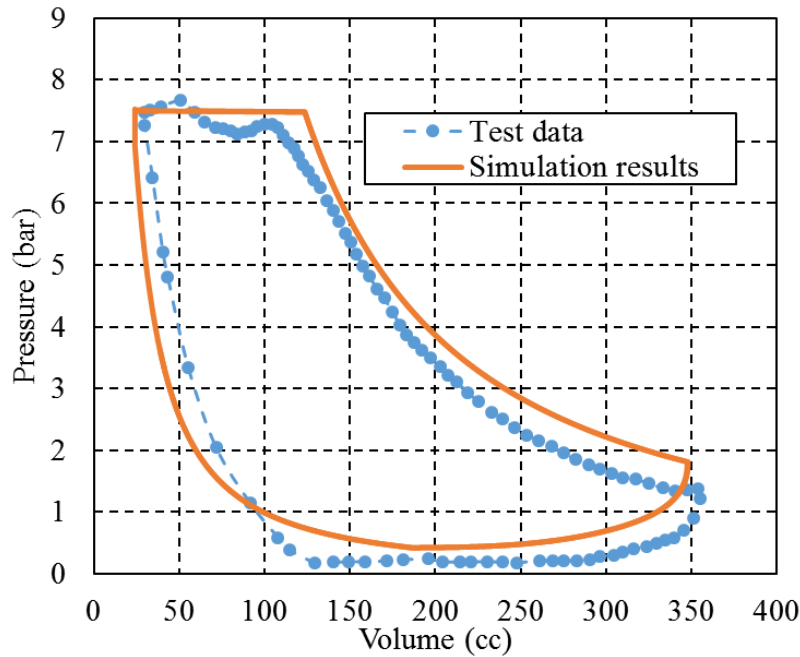
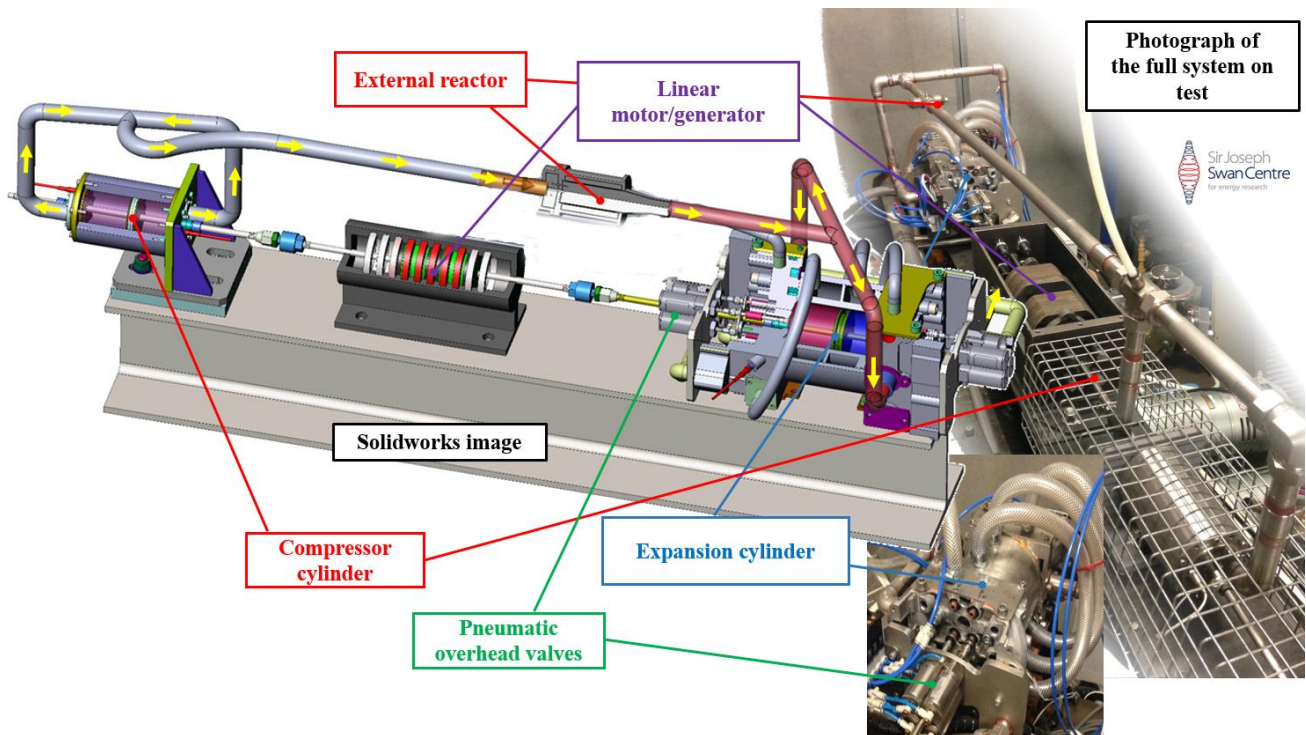


Figure 7. Comparison with test data from a Reciprocating Joule Engine [8]

### 4.3 Validation with LJEG prototype

The simulation model was also validated with the LJEG prototype developed at Newcastle University, which is comprised of a compressor, an expander, and an external heater. Two double-acting free-pistons are placed in the compressor (left) and the expander (right) respectively, which separates the cylinders into two opposite chambers. The figure of the prototype is shown in Figure 8, and more information can be found in elsewhere [9]. A control algorithm is developed in LabVIEW to set the valve timings with the piston displacement and velocity as the feedbacks. The bore of the expander is 80.0 mm, with a maximum stroke of 120.0 mm. The bore of the compressor is 66.0 mm, and the bore of the connection rod is 10.0 mm. The total moving mass of the system is 8.5 kg. The inlet pressure of

309 the expander is 2.5 bar during the testing. More details about the prototype and its configuration can  
310 be found elsewhere [3, 9].



311

312

Figure 8. LJEG prototype at Newcastle University

313 The validation results on the piston displacement and the cylinder pressure in the left chamber of the  
314 expander cylinder are presented in Figure 9 and Figure 10 respectively. It is found that the simulation  
315 model agrees with the piston movement in the tests, and the system operating frequency fits very well.  
316 The cylinder pressure profile in the expander can be precisely estimated during the compression and  
317 the expansion processes. There is a difference during the intake process as a simple step function is  
318 adopted to simulate the valve lifting profile, which cannot predict the gas pressure instantaneous  
319 fluctuations when the valves open and close. Despite these errors, the simulation model is considered  
320 to be of reasonable accuracy to estimate the operation characteristics of the LJEG system.

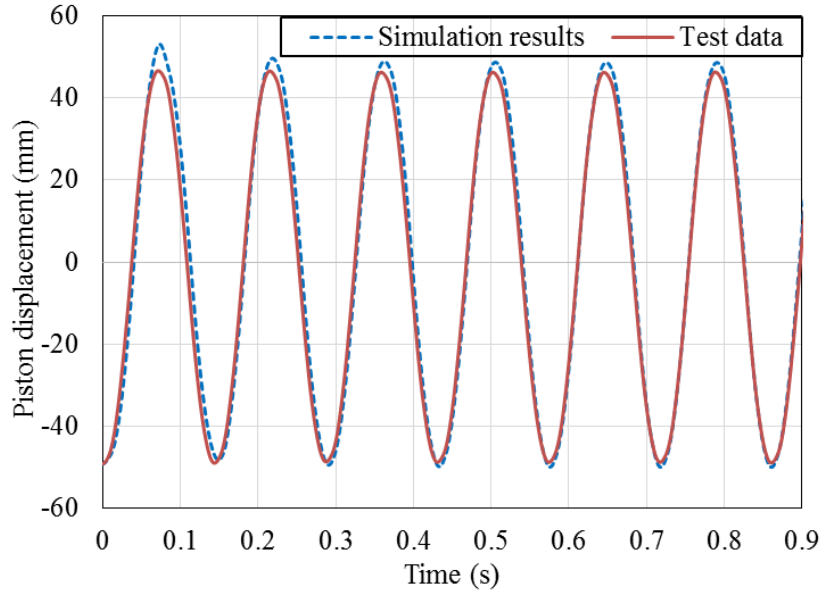


Figure 9. The comparison with test data from a LJEG prototype on piston displacement

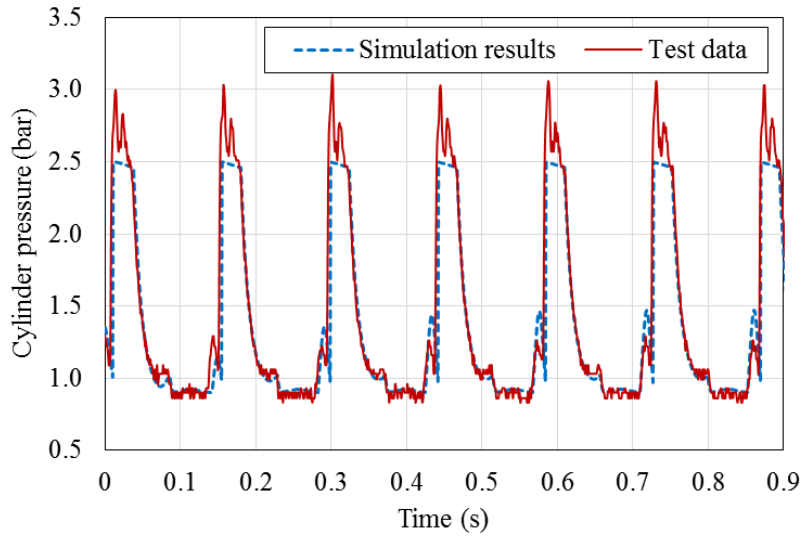


Figure 10. The comparison with the test data from the LJEG prototype on the cylinder pressure

## 5. Fundamental system performance

The values for the input variables during the current simulation are listed in Table 1. The inlet pressure of the expander is set to 7.0 bar, which is feasible for a compressor at the end of compression process.

The data in Table 3 shows the system performance with the input parameter shown in Table 1. The

indicated power from the linear expander is estimated to be 6582.0 W, and the indicated power from the linear compressor is estimated to be 1594.0 W. The electric power output can reach 4412.0 W. The engine thermal efficiency can reach above 34%, with an electric generating efficiency of 30% from our simulation [3, 9].

Table 3 LJEG system performance

Performance [Unit]	Value
Operation frequency [Hz]	15.0
Piston amplitude from central stroke [mm]	51.0
Clearance length [mm]	9.0
Peak piston velocity [m/s]	4.0
Compression ratio [-]	12.3

The piston displacement versus time is demonstrated in Figure 11, which shows certain similarity with a sinusoidal wave with a fixed amplitude and period during stable operation process after the beginning stage. The piston moves between its top dead centre (TDC) and bottom dead centre (BDC) from approximately -51.0 mm to +51.0 mm. The operation stroke is around 102.0 mm, and the clearance length is 9.0 mm, which can be adjusted by the valve timings, the inlet pressure of the expander, and the load of the generator. As there is no combustion in the expander and the driven pressure in the expander is lower (normally higher than 40 bar after combustion for an internal combustion engine), the clearance length is longer than that of an internal combustion free-piston engine.

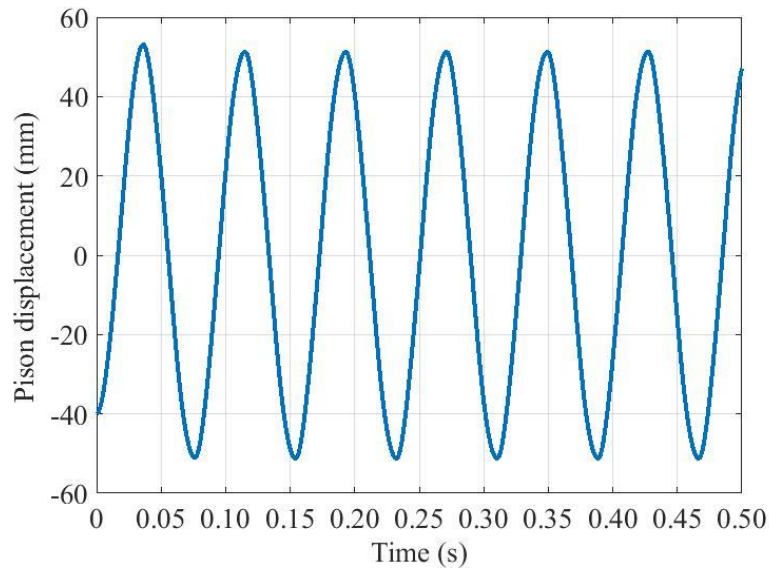


Figure 11. The piston displacement vs time

The piston velocity profile is demonstrated in Figure 12. As there is no combustion, the difference of the piston velocity during the gas intake process and the exhaust process is not significant. The piston velocity reaches its peak value before it crosses the midpoint of the stroke during the intake process. The peak piston velocity achieved is approximately 4.0 m/s, which is lower than that of a free-piston internal combustion engine with similar size (nearly 4.5 m/s) [31], due to a lower input pressure level without combustion. The corresponding system frequency is approximately 13 Hz (equivalent to 780 rpm) with the current operation conditions, which is also lower than the reported operation frequency of a free-piston internal combustion engine (20-50 Hz).



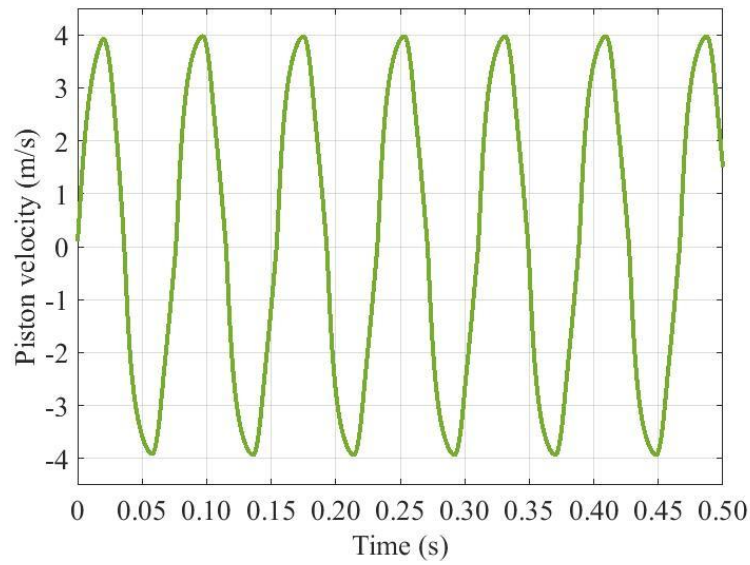


Figure 12. The piston velocity vs time

The pressure-displacement diagram of the left chamber of the linear expander is shown in Figure 13, with the valve open/closing timing marked on it. During the simulation, the intake valve is set to open when the piston reaches its TDC. The peak pressure in the expander is affected by the intake duration of the expander of the other side. When the intake duration of the other side is short, then the gas pressure at the end of compression process will be lower than the intake pressure, and vice versa.

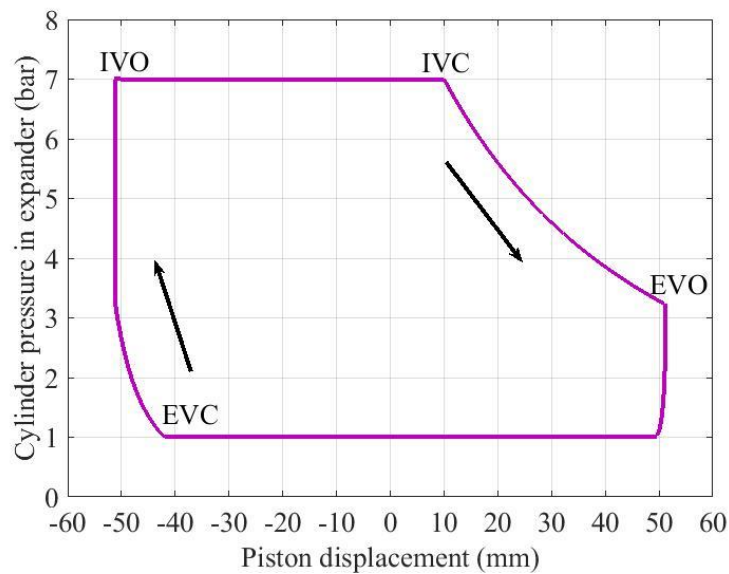
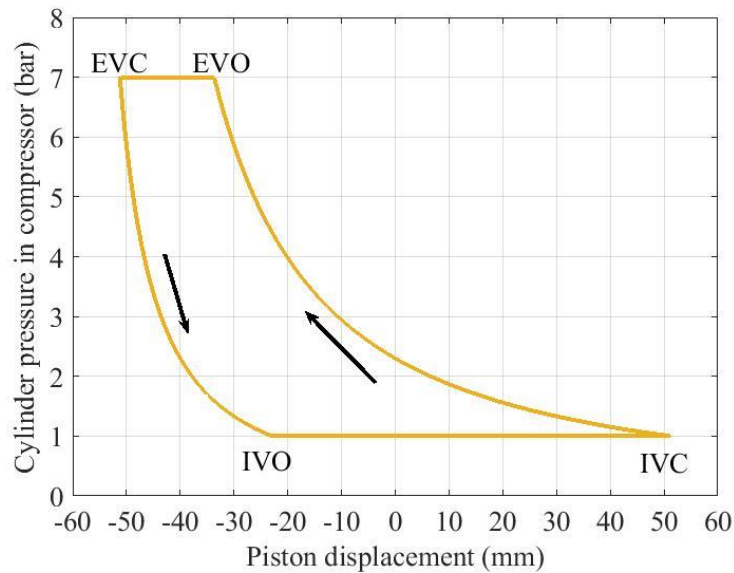


Figure 13. The pressure in the expander vs the piston displacement

361 The pressure in the left chamber of the linear compressor with piston displacement is shown in Figure  
 362 14, with the valve opening timing marked on it. The compression and expansion processes of the  
 363 compressor are assumed to be isentropic processes. Reed valves are employed in the LJEG prototype.  
 364 The inlet pressure for the linear compressor is equal with the ambient pressure, and the pressure in the  
 365 compressor is assumed to be drop to and maintain at the ambient pressure when the intake valve of the  
 366 compressor opens. The exhaust valve will be open when the gas pressure in the compressor reaches  
 367 the target pressure (7.0 bar in this simulation), and the compressor will then output the compressed gas  
 368 to the reactor for combustion with the fuel. The exhaust valve will be closed when the gas pressure in  
 369 the compressor drops below the target pressure.



370

371

Figure 14. Pressure in the compressor vs piston displacement

372

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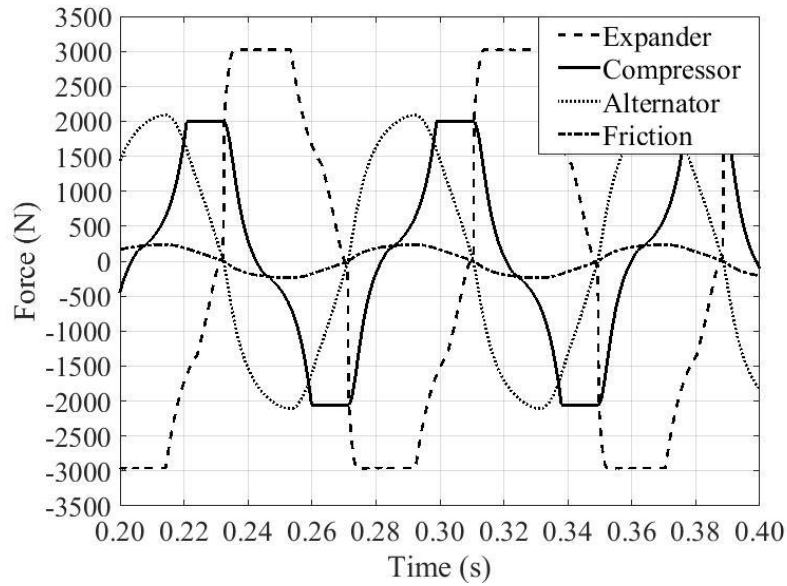
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377

The forces acting on the piston that contribute to the piston inertia force are compared in Figure 15. It  
 is found that the force from the expander is highest among all the forces acting on the piston, which  
 can reach up to 3500 N. The peak force from the generator is approximately 2100 N. The peak force  
 from the compressor is 2000 N, which is achieved at the end of the compression process, and stays at  
 the peak value during the outlet process. The force from the expander will overcome the forces from  
 the compressor, the linear generator, and the frictional force, and acts as an excite force to drive the

378 pistons reciprocate. As the force from the expander is generated by the gas pressure in its chambers,  
 379 the influence of the influence pressure will be significant to the system.



380  
 381 Figure 15 Forces vs time

382 The system power output with different system pressures (or the input pressure of the linear expander)  
 383 is shown in Figure 16, and all the other input parameters remained unchanged during the simulation.  
 384 Linear fittings for expander indicated power and electric power are presented in the same figure. It is  
 385 found that both the indicated power of the expander and the electric power of the linear alternator are  
 386 nearly in a linear relationship with the system pressure. When the system pressure is increased to above  
 387 7.5 bar, the electric power extracted from the LJEG system can be above 5.0 kW. As a result, with the  
 388 current setting of the system volumetric parameters and operating parameters, the indicated power of  
 389 the linear expander,  $P_{ex}$  (W) can be estimated by:

390 
$$P_{ex} = 1943.8 \times p_{in} - 6848.1 \quad (18)$$

391 The electric power output of the linear alternator,  $P_e$  (W) can be estimated by:

392 
$$P_e = 1247.4 \times p_{in} - 4214.9 \quad (19)$$

393 Where  $p_{in}$  (bar) is the inlet pressure of the linear expander, or the system pressure.

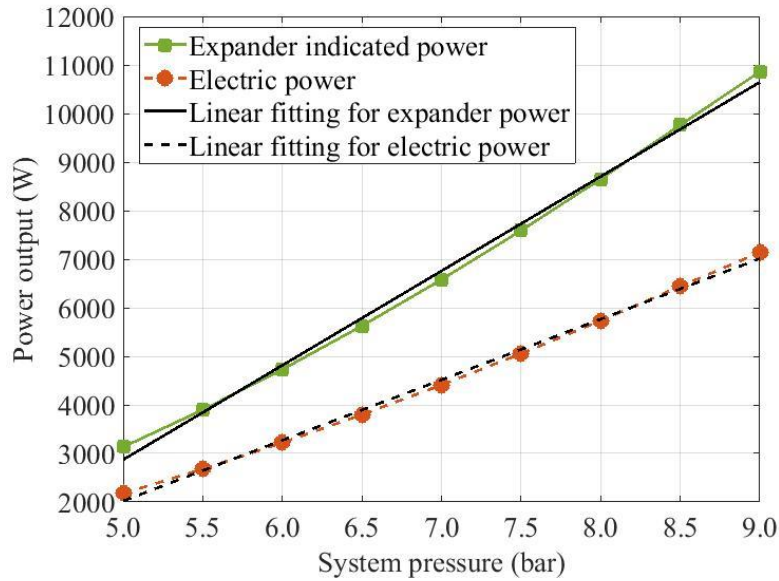


Figure 16. Power output with different system pressures

## Conclusions

In this research, the background and recent development of the LJEG was summarised. A detailed numerical model was described, and model validation was performed with test data from both a reciprocating Joule Engine and a LJEG prototype. Fundamental system operation characteristics were presented. The main conclusions from this work are listed below:

- (1) It was found that the piston displacement shows certain similarity with a sinusoidal wave with fixed amplitude and period. The operation stroke is around 102.0 mm, and the clearance length is 9.0 mm.
- (2) The peak piston velocity and system operation frequency are found to be lower than that of a free-piston internal combustion engine with similar size, due to a lower input pressure level without combustion. The peak piston velocity achieved is approximately 4 m/s, and the corresponding system frequency is approximately 13 Hz (equivalent to 780 rpm) with the current operation conditions.
- (3) The electric power output can reach 4.4 kW<sub>e</sub>, the engine thermal efficiency can reach above 34%, with an electric generating efficiency of 30%.

409 (4) The peak pressure in the expander is affected by the intake duration of the expander of the other  
410 side. When intake duration of the other side is short, then the gas pressure at the end of compression  
411 process will be lower than the intake pressure, and vice versa.

412 (5) Both the indicated power of the expander and the electric power of the linear alternator are nearly  
413 in a linear relationship with the system pressure.

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